SOLUTIONS OF PROBLEMS.

489 (Algebra). Proposed by S. A. COREY, Albia, Iowa.

Prove or disprove the following:

$$\begin{vmatrix} -x & -ay & -bu & abv \\ y & x & -bv & -bu \\ u & av & x & ay \\ -v & -u & y & -x \end{vmatrix}^2 + a\begin{vmatrix} x & -x & -bu & abv \\ y & y & -bv & -bu \\ u & u & x & ay \\ v & -v & y & -x \end{vmatrix}^2$$

$$+ b\begin{vmatrix} x & -ay & -x & abv \\ y & x & y & -bu \\ u & av & u & ay \\ v & -u & -v & -x \end{vmatrix}^2 + ab\begin{vmatrix} x & -ay & -bu & -x \\ y & x & -bv & y \\ u & av & x & u \\ v & -u & y & -v \end{vmatrix}^2 = \begin{vmatrix} x & -ay & -bu & abv \\ y & x & -bv & -bu \\ u & av & x & ay \\ v & -u & y & -x \end{vmatrix}^2.$$

II. Solution by A. M. Harding, University of Arkansas.

The quantities X, Y, U, V, W, defined by the equations

$$\begin{aligned} xX - ayY - buU + abvV &= -xW, \\ yX + xY - bvU - buV &= yW, \\ uX + avY + xU + ayV &= uW, \\ vX - uY + yU - xV &= -vW, \end{aligned}$$

are proportional to the five determinants taken in order.

It can be easily shown that the last determinant in the left member of the proposed equation is equal to zero for all values of x, y, u, v, a, b. Hence the above equations may be written in the form

$$\begin{aligned} (X + W)x - aYy - bUu &= 0, \\ Yx + (X - W)y - bUv &= 0, \\ Ux + (X - W)u + aYv &= 0, \\ Uy - Yu + (X + W)v &= 0. \end{aligned}$$

The quantities x, y, u, v, can satisfy this system of homogeneous linear equations if, and only if, the determinant

$$\begin{vmatrix} X + W & -aY & -bU & 0 \\ Y & X - W & 0 & -bU \\ U & 0 & X - W & aY \\ 0 & U & -Y & X + W \end{vmatrix}$$

is equal to zero. It can be shown that the value of this determinant is k^2, where

$$k = (X^2 - W^2) + aY^2 + bU^2.$$

Hence $X^2 + aY^2 + bU^2 = W^2$, or, since $V = 0$, $X^2 + aY^2 + bU^2 + abV^2 = W^2$. That is, the relation stated in the problem holds for all values of x, y, u, v, a, b.

Note. We are publishing a second solution of this problem for two reasons: First, because the solution above is essentially different from the one published in the March number; and second, because the conclusion drawn in that solution that the identity does not always exist is incorrect. Editors.

271 (Number Theory). Proposed by HORACE OLSON, Chicago, Illinois.

Prove that if x, y, z, u, v, and w are integers such that $x^2 + y^2 = u^2$, $x^2 + z^2 = v^2$, $y^2 + z^2 = w^2$, then the product $xyzuvw$ is divisible by 518400.